Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WFM03/01**

# Mathematics

## International Advanced Subsidiary/ Advanced Level
## Further Pure Mathematics F3

**You must have:**
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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Pearson

**1.** Given that

$$y = 3x \arcsin 2x \qquad 0 \leqslant x \leqslant \frac{1}{2}$$

(a) determine an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

**(2)**

(b) Hence determine the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$, giving your answer in the form $a\pi + b$ where $a$ and $b$ are fully simplified constants to be found.

**(1)**

**Question 1 continued**

**(Total for Question 1 is 3 marks)**

**2.** A hyperbola $H$ has equation

$$\frac{x^2}{a^2} - \frac{y^2}{5} = 1 \quad \text{where } a \text{ is a positive constant}$$

The line with equation $x = \frac{4}{3}$ is a directrix of $H$

(a) Write down an equation of the other directrix.

**(1)**

(b) Determine

(i) the value of $a$

(ii) the coordinates of each of the foci of $H$

**(5)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.** Solve the equation

$$4\tanh x - \operatorname{sech} x = 1$$

giving your answer in the form $x = \ln k$ where $k$ is a fully simplified rational number.

**(6)**

**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4.** (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}} \, dx$$

**(2)**

(b) Hence determine the exact value of

$$\int_{-2}^{2} \frac{1}{\sqrt{9x^2 + 16}} \, dx$$

Give your answer in the form $a \ln(b + c\sqrt{13})$, where $a$, $b$ and $c$ are rational numbers.

**(3)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 5 marks)**

**5.**

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix}$$ where $a$ is a positive constant

(a) Determine the exact value of $a$ for which the matrix $\mathbf{A}$ is singular.

**(2)**

Given that 2 is an eigenvalue of $\mathbf{A}$

(b) determine

(i) the value of $a$

(ii) the other two eigenvalues of $\mathbf{A}$

**(5)**

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(c) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

**(5)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 12 marks)**

**6.** A curve has parametric equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

where $a$ is a positive constant.

(a) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = ka^2 \sin^2 \frac{\theta}{2}$$

where $k$ is a constant to be determined.

**(4)**

The part of the curve from $\theta = 0$ to $\theta = 2\pi$ is rotated through $2\pi$ radians about the $x$-axis.

(b) Determine the area of the surface generated, giving your answer in terms of $\pi$ and $a$.

[*Solutions relying on calculator technology are not acceptable*.]

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 9 marks)**

**7.** The plane $\Pi$ has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where $\lambda$ and $\mu$ are scalar parameters.

(a) Determine a vector perpendicular to $\Pi$

**(2)**

The line $l$ meets $\Pi$ at the point (1, 2, 3) and passes through the point (1, 0, 1)

(b) Determine the size of the acute angle between $\Pi$ and $l$

Give your answer to the nearest degree.

**(4)**

(c) Determine the shortest distance between $\Pi$ and the point (6, −3, −6)

**(4)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 10 marks)**

**8.**

$$I_n = \int \cos^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$$

**(4)**

(b) Show that for positive even integers $n$

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)...5 \times 3 \times 1}{n(n-2)(n-4)...6 \times 4 \times 2} \times \frac{\pi}{2}$$

**(4)**

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, dx$$

**(3)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 11 marks)**

**9.** The ellipse $E$ has equation

$$x^2 + 9y^2 = 9$$

The foci of $E$ are $F_1$ and $F_2$

(a) (i) Determine the coordinates of $F_1$ and the coordinates of $F_2$

(ii) Write down the equation of each of the directrices of $E$

**(4)**

The point $P$ lies on the ellipse.

(b) Show that $|PF_1| + |PF_2| = 6$

**(3)**

The straight line through $P$ with equation $y = 2x + c$ meets $E$ again at the point $Q$

The point $M$ is the midpoint of $PQ$

(c) Show that as $P$ varies the locus of $M$ is a straight line passing through the origin.

**(6)**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 13 marks)**

**TOTAL FOR PAPER IS 75 MARKS**